SCIENTIFIC LETTERS

Econometrics Letters Volume (1), 1, 2014, June.

Nonlinearity and Smooth Breaks in Unit Root Testing

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Abstract

We develop unit root tests for time series where the alternative is unit-root tests that allow under the alternative hypothesis for a smooth transition between deterministic linear trends, around which stationary asymmetric adjustment may occur due to exponential smooth transition auto-regression (ESTAR), and provide their small sample properties. We apply our tests for investigating PPP hypothesis in the G7 sample.

Keywords: Smooth Break; Nonlinear Unit Root Test; PPP.
JEL Codes: C12; C22; O47.

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1. Introduction

In this study we have developed a unit root test by combining Kapetanois et al (2003) (henceforth KSS) and Leybourne, Newbold and Vougas (1998) (henceforth LNV). KSS (2003) employ exponential smooth transition autoregressive (ESTAR) models to propose tests of the null hypothesis of a unit root that allow under the alternative hypothesis for stationary nonlinear adjustment towards a fixed mean. Thus, we extent the KSS tests to the case of a nonlinear attractor[3].

Section 2 of this paper develops the proposed test statistics and represents their critical values. Section 3 provides the small sample performance of proposed test in comparison with the power of the ADF, LNV, Sollis, KSS and EG tests. Section 4 presents the application of our aforementioned tests to PPP hypothesis.

2. The model and testing framework

Let y_t be a changing trend function with smooth transition on the time domain $t = 1, 2, ..., T$.

$$y_t = \alpha + \alpha_2 S_t(\gamma,\tau) + \varepsilon_t \qquad (1)$$

$$y_t = \alpha + \beta_1 t + \alpha_2 S_t(\gamma,\tau) + \varepsilon_t \qquad (2)$$

$$y_t = \alpha + \beta_1 t + \alpha_2 S_t(\gamma,\tau) + \beta_2 t S_t(\gamma,\tau) + \varepsilon_t \qquad (3)$$

where ε_t is a zero mean $I(0)$ process and $S_t(\gamma,\tau)$ is logistic smooth transition function, based on a sample of size T and N,

$$S_t(\gamma,\tau) = \left[1 + \exp\{-\gamma(t - \tau T)\}\right]^{-1}, \quad \gamma > 0 \qquad (4)$$

In this modeling strategy, the structural change is modeled as smooth transition between different regimes rather than instantaneous structural break Leybourtne *et al.* (1996). The transition function $S_t(\gamma,\tau)$ is continuous function bounded between 1 and 0. Thus the STR model can be interpreted as regime-switching model that allows for two regimes, associated with the extreme values of the transition function, $S_t(\gamma,\tau) = 0$ and $S_t(\gamma,\tau) = 1$, whereas the transition from one regime to the other is gradual. The parameter γ determines the smoothness of the transition, and thus, the smoothness of transition from one regime to the other. The two regimes are associated with small and large values of the transition variable $s_t = t$ relative to the threshold $c = \tau$. For the large values of γ, $S_t(\gamma,\tau)$ passes through the interval $(0,1)$ very rapidly, and as γ approaches $+\infty$ this function changes value from 0 to 1 instantaneously at time $t = \tau T$. Therefore, if we assume that ε_t is zero mean $I(0)$ process, and then model 1 y_t is stationary process around a mean which changes from initial value α_1 to final value $\alpha_1 + \alpha_2$. Leybourne *et al.* (1996) also give similar conditions for model 2 and 3. In these specifications no change and one instantaneous structural change are limiting cases whereas this specification is more general which it covers gradual structural changes as well[4].

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[3] Enders and Granger (1998) proposed unit root test for two regime TAR model. They named as the linear trend as linear attractor. Hence, following their suggestion we called this nonlinear trend as nonlinear attractor.

[4] For further discussion and the possible extension see Leybourne *et al.* (1996).

We establish the hypotheses for unit root testing based on equation 1, 2 and 3 as follows:

$H_0 : Unit \quad Root$, (Linear Nonstationary)

$H_a : Nonlinear \quad Stationary$ (Nonlinear and Stationary around smoothly changing trend and intercept) (5)

Following Leybourne *et al.* (1996) the test statistics proposed here is calculated with a two-step procedure:

Step 1. Using a nonlinear least squares (NLS) algorithm, estimate only deterministic component of the preferred model and compute the NLS residuals

Model 1 $\hat{\varepsilon}_t = y_t - \hat{\alpha}_1 - \hat{\alpha}_2 S_t(\gamma, \tau)$

Model 2 $\hat{\varepsilon}_t = y_t - \hat{\alpha}_1 - \hat{\beta}_1 t - \hat{\alpha}_2 S_t(\gamma, \tau)$

Model 3 $\hat{\varepsilon}_t = y_t - \hat{\alpha}_1 + \hat{\beta}_1 t + \hat{\alpha}_2 S_t(\gamma, \tau) + \hat{\beta}_2 t S_t(\gamma, \tau)$

Step 2. Compute the KSS statistic, the t ratio associated with $\hat{\rho}_i$ in the ordinary least squares (OLS) regression

$$\Delta\hat{\varepsilon}_t = \hat{\rho}\hat{\varepsilon}_t^3 + \sum_{j=1}^{k} \hat{\delta}_j \Delta\hat{\varepsilon}_{t-j} + \hat{\eta}_t \qquad (6)$$

For model 1, 2 and 3 we demonstrate t statistics for $\hat{\rho}_i$ as $\overline{t}_{br1}$, $\overline{t}_{br2}$, and $\overline{t}_{br3}$, respectively.

$H_0 : \rho = 0$, for all i, (Linear Nonstationary)

$H_0 : \rho < 0$, for some i, (Nonlinear and Stationary around nonlinear trend and intercept)

Table 1. Critical Values

	Model 1			**Model 2**			**Model 3**		
	%10	%5	%1	%10	%5	%1	%10	%5	%1
25	-3.691	-4.133	-5.056	-4.296	-4.728	-5.543	-4.609	-5.048	-5.873
50	-3.521	-3.870	-4.571	-3.963	-4.327	-5.106	-4.214	-4.593	-5.380
100	-3.509	-3.821	-4.443	-3.889	-4.202	-4.777	-4.090	-4.411	-5.041
200	-3.496	-3.810	-4.424	-3.885	-4.189	-4.771	-4.062	-4.382	-4.980
500	-3.489	-3.801	-4.412	-3.879	-4.180	-4.757	-4.053	-4.370	-4.969

2.1 Finite sample performance

We have investigated the empirical power of the test by using the below data generating process where the process is stationary nonlinear adjustment around a smooth transition from one constant value to another. The following ST-ESTAR(1) was employed as a DGP.

$$y_t = \alpha_1 + \alpha_2 S_t(\gamma, \tau) + \varepsilon_t \ , \quad \mu_0 = 0$$

$$\Delta\varepsilon_t = \alpha + \gamma\varepsilon_{t-1}\left(1 - \exp\left[-\theta\varepsilon_{t-1}^2\right]\right) + \eta_t \ , \qquad \eta_t \sim NID(0,1)$$

where S_t is defined as before, and all combinations of the following parameter values were used; two extreme values for gamma parameter $\gamma = -0.1, -1.0$, for the transition speed again ve use two extreme values $\theta = 0.01, 1.0$ and for the structural break we use small and large structural break parameters $\alpha_2 = 2.0, 10.0$. The results from these power experiments for a sample size of $T = 100$ are given in Table 2.1.1.

Table2.1.1 The power comparison of alternative tests

α_2	λ	c	θ	γ	$S_{\alpha,NL}$	s_α	t_{s_α}	F_α	t_{NL}	$T_{\max_t}$	Φ_t	τ_μ
2.0	0.5	0.2	0.01	-0.1	0.062	0.046	0.046	0.034	0.062	0.064	0.048	0.044
2.0	0.5	0.2	1.0	-0.1	0.086	0.076	0.070	0.068	0.190	0.144	0.162	0.096
2.0	0.5	0.2	0.01	-1.0	0.164	0.130	0.148	0.116	0.234	0.176	0.208	0.188
2.0	0.5	0.2	1.0	-1.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.0	0.5	0.5	0.01	-0.1	0.070	0.064	0.064	0.052	0.086	0.080	0.074	0.040
2.0	0.5	0.5	1.0	-0.1	0.100	0.092	0.084	0.086	0.148	0.110	0.134	0.100
2.0	0.5	0.5	0.01	-1.0	0.152	0.132	0.110	0.108	0.208	0.152	0.180	0.152
2.0	0.5	0.5	1.0	-1.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.0	5.0	0.2	0.01	-0.1	0.050	0.038	0.042	0.026	0.078	0.068	0.068	0.040
2.0	5.0	0.2	1.0	-0.1	0.116	0.110	0.092	0.098	0.206	0.142	0.174	0.100
2.0	5.0	0.2	0.01	-1.0	0.160	0.130	0.126	0.124	0.204	0.174	0.190	0.142
2.0	5.0	0.2	1.0	-1.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.0	5.0	0.5	0.01	-0.1	0.052	0.042	0.040	0.032	0.092	0.060	0.074	0.060
2.0	5.0	0.5	1.0	-0.1	0.096	0.118	0.088	0.098	0.200	0.120	0.164	0.082
2.0	5.0	0.5	0.01	-1.0	0.190	0.162	0.130	0.150	0.236	0.164	0.212	0.174
2.0	5.0	0.5	1.0	-1.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
10.0	0.5	0.2	0.01	-0.1	0.026	0.012	0.016	0.008	0.006	0.020	0.002	0.018
10.0	0.5	0.2	1.0	-0.1	0.076	0.068	0.052	0.054	0.006	0.020	0.000	0.022
10.0	0.5	0.2	0.01	-1.0	0.120	0.080	0.060	0.074	0.016	0.026	0.012	0.024
10.0	0.5	0.2	1.0	-1.0	1.000	1.000	1.000	1.000	0.008	0.038	0.002	0.231
10.0	0.5	0.5	0.01	-0.1	0.042	0.042	0.040	0.034	0.014	0.010	0.004	0.014
10.0	0.5	0.5	1.0	-0.1	0.114	0.088	0.068	0.072	0.014	0.012	0.004	0.012
10.0	0.5	0.5	0.01	-1.0	0.242	0.161	0.191	0.111	0.030	0.020	0.010	0.030
10.0	0.5	0.5	1.0	-1.0	1.000	1.000	1.000	1.000	0.072	0.058	0.042	0.318
10.0	5.0	0.2	0.01	-0.1	0.114	0.028	0.034	0.016	0.010	0.032	0.002	0.066
10.0	5.0	0.2	1.0	-0.1	0.144	0.074	0.068	0.068	0.004	0.028	0.004	0.076
10.0	5.0	0.2	0.01	-1.0	0.188	0.070	0.086	0.064	0.004	0.024	0.000	0.064
10.0	5.0	0.2	1.0	-1.0	1.000	1.000	1.000	1.000	0.004	0.036	0.002	0.434
10.0	5.0	0.5	0.01	-0.1	0.160	0.060	0.096	0.062	0.026	0.032	0.018	0.064
10.0	5.0	0.5	1.0	-0.1	0.274	0.148	0.180	0.144	0.016	0.024	0.008	0.052
10.0	5.0	0.5	0.01	-1.0	0.398	0.190	0.198	0.182	0.020	0.032	0.014	0.068
10.0	5.0	0.5	1.0	-1.0	1.000	1.000	1.000	1.000	0.058	0.070	0.038	0.282

Note 1: Leybourne *et al.* (1996) stated that Model 1's natural competitor is intercept plus trend terms for ADF test. Therefore, we compare the Model 1 with the intercept plus trend term with the other tests namely KSS, ADF and EG.

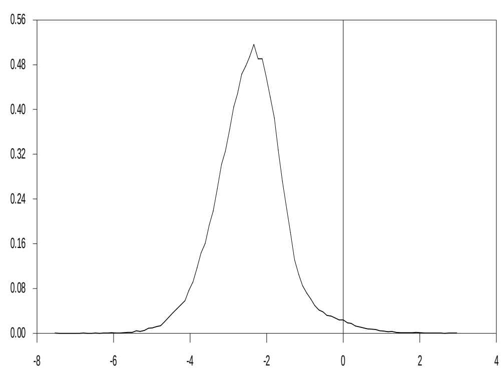
Note 2 : $S_{\alpha,NL}$, s_α, t_{s_α} and F_α denotes the proposed test, LNV, Solis max-t and F tests, respectively. t_{NL}, $T_{\max_t}$, Φ_t and τ_μ denotes the KSS, EG max-t, EG F and DF tests, respectively. The second group of tests does not cover the structural break in their testing procedure.

For a small break ($\alpha_2 = 2.0$), the power of KSS test exceeds that of newly proposed test. However, as expected in the large break ($\alpha_2 = 10.0$) the newly proposed test over performs all of the tests in all parameter regions.

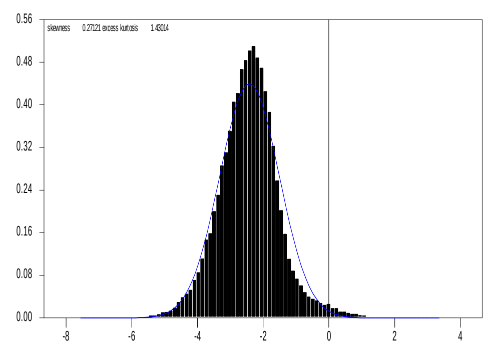
2.2 The Asymptotic Properties of the Proposed Test

Leybourne et al. (1996) state that as NLLS estimation of the parameters gamma and tou does not admit closed form solutions, hence, it would be extremely difficult to subsequently establish any analytical relationship between the residual terms which is obtained from STR estimation of deterministic component and the dependent variable. Therefore, this makes determination of the null asymptotic distribution of the test statistics by analytical means more or less intractable. Moreover in our testing procedure we are introducing an other nonlinearity around the deterministic component which it makes more harder to obtained the asymptotic distribution. Thus, we use simulation methodology to see the nonlinear asymptotic relationship. In order to see the difference between normal distribution and the distribution that we obtained from our test statistics, we have obtained density function of the test statistics. The below figures are the simulation results:

The simulated density of test statistics for T= 50



The comparison with normal distribution



Observations	50000
Sample Mean	-2.415
Variance	0.822
Standard Error	0.906
Skewness	0.271 (0.000)
Kurtosis (excess)	1.430 (0.000)
Jarque-Bera	4874.005 (0.000)

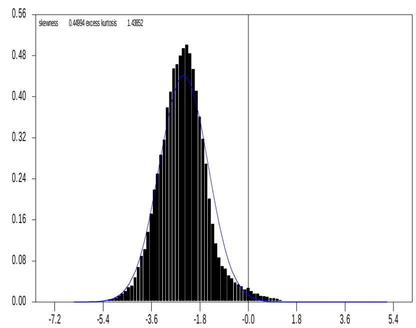
*The solid line is normal distribution and the shaded area is simulated density function of the proposed test

* Number of Simulation 50000

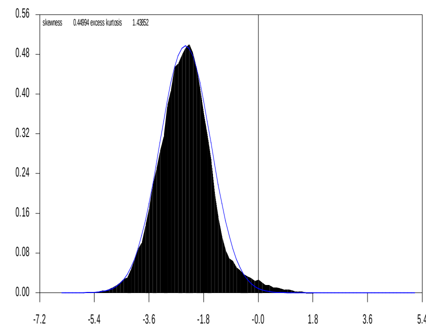
* Time series dimension T = 50

Figure 1. The simulated density of test statistics and comparison with normal distribution for T: 50

The simulated density of test statistics



The comparison with normal distribution



Sample Mean	-2.400
Variance	0.815
Standard Error	0.903
Skewness	0.449 (0.000)
Kurtosis (excess)	1.438 (0.000)
Jarque-Bera	5998.221 (0.000)

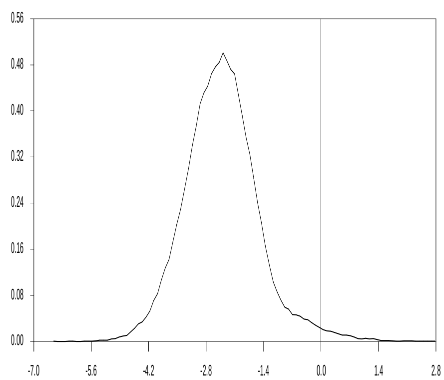
*The solid line is normal distribution and the shaded area is simulated density function of the proposed test

* Number of Simulation 50000

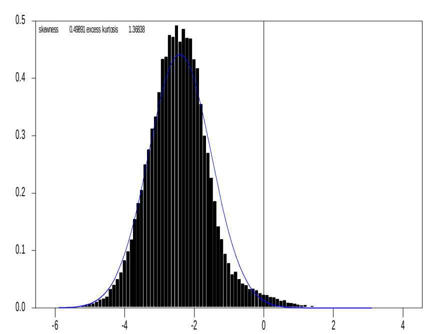
* Time series dimension T = 100

Figure 2. The simulated density of test statistics and comparison with normal distribution for T: 100

The simulated density of test statistics for



The comparison with normal distribution



Sample Mean	-2.413
Variance	0.821
Standard Error	0.906
Skewness	0.506 (0.000)
Kurtosis (excess)	1.408 (0.000)
Jarque-Bera	6271.965 (0.000)

*The solid line is normal distribution and the shaded area is simulated density function of the proposed test

* Number of Simulation 50000

* Time series dimension T = 500

Figure 3. The simulated density of test statistics and comparison with normal distribution for T: 500

3. Empirical Example

In this section we empirically apply all the unit root tests in power analysis to examine the validity of the purchasing power parity (PPP) hypothesis for Argentina over the period 2003:6-2011:10. Monthly data on bilateral exchange rate of the national currency against the U.S. dollar and on consumer price indices (CPI) was taken from International Monetary Fund's *International Financial Statistics* (IFS) database. The base year for the CPI is 1997. All variables were put into natural logarithms before the analysis.

Table 11. The PPP hypothesis under different unit root tests.

	$S_{\alpha,NL}$	s_{α}	F_{α}	t_{NL}	Φ_t	τ_{μ}
Argentina	-7.315	-2.156	2.325	-2.398	1.813	-1.770

Note: LNV %10 %5 and %1 significance level -3.909, -4.232,-4.882, Sollis %10 %5 and %1 significance level 7.844, 9.191,12.244. KKS %10 %5 and %1 significance level -2.66, -2.93,-3.48. EG %10 %5 and %1 significance level 3.79, 4.64 ,6.57. ADF 10 %5 and %1 significance level -2.58, -2.89, -3.51

The results of the ADF, PP, KSS, EG and Sollis unit root tests recommend that the null hypothesis of unit root is rejected at conventional significance levels. These results are contradicting to the PPP hypothesis. On the other hand, our newly proposed test that allows for nonlinear adjustment towards LNV type trend function rejects the null hypothesis of unit root at 1% significance level which provides an evidence for PPP hypothesis. This finding recommends that a model that allows for gradual structural breaks and nonlinear adjustment might be more suitable for the Argentina RER series.

Figure 1. Estimated STR type trend functions (Model A) for Argentina

4. Conclusion

In this study, we have proposed a nonlinear unit root tests which also considers structural break. By using this newly proposed test we show the validity of PPP hypothesis for the Argentina real exchange rate series.

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